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kinds ; and if 1,000 equal risks of the separate kinds were at first undertaken by 1,000 different Offices—it is manifest that if each of these 1,000 Offices, instead of confining themselves to the same kinds of risks, exchanged, in the second place, one of its risks with each of the other 999, so as to have all heterogeneous instead of all homogeneous risks, the profits and losses of each Office being supposed to be fairly balanced on the original supposition, they would remain equally balanced under the new distribution, nor is there anything in the supposed change of combination that could affect the liability to greater or less deviations : at least, if there were any accident that could lead to such an inconvenience, it would probably be rather more likely to be diminished by the combination of heterogeneous elements than by the confining the separate Societies to their primitive homogeneous undertakings, which might possibly partake more of the nature of an undivided single risk in the liability to inconvenient fluctuations.

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*On a Method of Distributing the Surplus among the Assured in a Life Assurance Office. By T. B. SPRAGUE, M.A., Fellow of St. John's College, Cambridge.*

SUPPOSE that the portion of surplus allotted to a particular policy at the division of profits, is applied so as to relieve the assured from the annual payments after arriving at an age to be determined by the amount of that portion or cash bonus, the other conditions of the policy remaining the same ; or again, suppose that the cash bonus is so applied as to convert the assurance into an endowment assurance, payable at an age to be determined :—it is proposed to investigate formulæ to apply to these two cases.

In the first case, the cash bonus must be equal to the present value at the time of division of profits of a deferred annuity, of which the annual payments are equal to the premium for the assurance, the number of years for which the annuity is deferred being unknown ; and the problem, in fact, is to find the number of years.

Let  $m$  be the age of the life assured at the time of the division of profits ;  $P$ , the annual premium ;  $C$ , the cash bonus ;  $x$ , the number of years for which the annuity is deferred, which is at pre-

sent unknown. Then, by the usual formula,  $C = P \cdot \frac{N_{m+x}}{D_m}$ . Multiplying by  $D_m$  and dividing by  $P$ ,  $N_{m+x} = \frac{C}{P} \cdot D_m$ .

In this equation the second member is completely known: then, calculating its value, we look down the  $N$  column, and simple inspection shows which value of  $N$  comes nearest to the calculated value; consequently the value of  $x$  is known, and the age at which the payments are to cease.

In the case of a second cash bonus  $C'$  after an interval of say five years being applied in the same way, so that the annual payments shall now cease at the age  $y$ , we have to take the value of a *deferred temporary annuity* equivalent to  $C'$ . The formula in this case is  $C' = P \cdot \frac{N_y - N_{m+x}}{D_{m+5}}$ ; and we have for  $y$ ,  $N_y = N_{m+x} + \frac{C'}{P} \cdot D_{m+5}$ , whence we find  $y$  by the same process as  $x$  in the former case.

For the second part of the problem we proceed in the following manner:—The value of £1 to be paid on the life  $m$  attaining  $m+x$  years, or at previous death, is  $\frac{M_m - M_{m+x} + D_{m+x}}{D_m}$ . The value of

£1 to be paid on the death of the same life is  $\frac{M_m}{D_m}$ : the difference

between these two is the benefit the life assured for £1 derives by having his assurance paid on his attaining the age  $m+x$  or at previous death, instead of at death absolutely. This difference =

$\frac{D_{m+x} - M_{m+x}}{D_m}$ . But this is not the whole benefit; for his assurance being converted into an endowment assurance payable at the

age  $m+x$ , of course all the annual payments will cease after  $x-1$

years; the further benefit that this amounts to is  $P \cdot \frac{N_{m+x-1}}{D_m}$ . the

amount assured being  $S$ , the whole benefit will be

$$S \cdot \frac{D_{m+x} - M_{m+x}}{D_m} + P \cdot \frac{N_{m+x-1}}{D_m},$$

which by hypothesis is equal to  $C$ . Hence  $S(D_{m+x} - M_{m+x}) + P \cdot N_{m+x-1} = C \cdot D_m$ .

The only way of solving this equation will be a tentative one; calculating the value of the first member for several ages, and finding which of them comes nearest to the second member, a process which will no doubt be somewhat troublesome.\*

\* We observe that one Office in its advertisements and prospectus proposes to divide its profits in this manner. It would be interesting to know the methods pursued for that purpose.